EMPIRICAL CRYPTO ASSET PRICING USING FACTOR MODELS WITH HIGH-DIMENSIONAL CHARACTERISTICS

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Consider a dynamic latent factor model with linear loadings

\[ r_{i,t+1} = z_{i,t}^\top \Gamma \beta_{i,t} f_{t+1} + \epsilon_{i,t+1}, \quad \mathbb{E}[\epsilon_{i,t+1}|z_{i,t}] = 0, \]

where we observe, for assets \( i \) and time periods \( t \),

- asset excess returns \( r_{i,t+1} \in \mathbb{R} \) and
- asset characteristics \( z_{i,t} \in \mathbb{R}^p \).
PREVIEW: MAIN THEORY CONTRIBUTIONS

In this setup, under the novel asymptotics of $p, T, N \to \infty$, contribute a new estimation procedure for

- latent loadings $\Gamma_{\beta} \in \mathbb{R}^{p \times k}$ and
- latent factors $f_{t+1} \in \mathbb{R}^{k}$, for all $t$;

and, prove the consistency of these estimators.

Also, I extend to this setting a classic asset pricing test and provide an asymptotically valid inference procedure.
Broadly: Study the dynamics of crypto asset returns.

Specifically:

- Measuring expected returns through lens of factor models.
- What characteristics are the drivers of returns?
- What is the inflation risk premium in the crypto asset class?
- If we relax interpretability, what is the maximum out-of-sample predictability that can we achieve?
Static observable factor model:

\[ r_{i,t+1} = \beta_i \top f_{t+1} + \epsilon_{i,t+1} \]

(\(NT + Tk\) data \(\gtrsim\) \(Nk\) params.)
MOTIVATION

Static observable factor model:

\[ r_{i,t+1} = \beta_i^\top f_{t+1} + \epsilon_{i,t+1} \]

\((NT + Tk)\) data \(\succeq (Nk)\) params.

Static latent factor model:

\[ r_{i,t+1} = \beta_i^\top f_{t+1} + \epsilon_{i,t+1} \]

\(NT \succeq Nk + Tk\)
MOTIVATION

Static observable factor model:

\[ r_{i,t+1} = \beta_i^T f_{t+1} + \epsilon_{i,t+1} \]

\((NT + Tk)\) data \(\gtrsim (Nk)\) params.

Static latent factor model:

\[ r_{i,t+1} = \beta_i^T f_{t+1} + \epsilon_{i,t+1} \]

\(NT \gtrsim Nk + Tk\)

Dynamic latent factor model:

\[ r_{i,t+1} = z_{i,t}^T \Gamma_\beta f_{t+1} + \epsilon_{i,t+1} \]

\(NT(1 + p) \gtrsim pk + Tk\)
MOTIVATION

Static observable factor model: \( r_{i,t+1} = \beta_i^\top f_{t+1} + \epsilon_{i,t+1} \).

Static latent factor model: \( r_{i,t+1} = \beta_i^\top f_{t+1} + \epsilon_{i,t+1} \).

Dynamic latent factor model: \( r_{i,t+1} = z_{i,t}^\top \Gamma \beta f_{t+1} + \epsilon_{i,t+1} \).

Nonparametric dynamic latent factor model:

\[
r_{i,t+1} = f(z_{i,t})^\top g(r_{i,t}) + \epsilon_{i,t+1}
\]
**SETUP**

Assume for time periods $t = 1, \ldots, T$ and assets $i = 1, \ldots, N$, we observe realizations of random variables

- asset excess returns $r_{i,t+1} \in \mathbb{R}$ and

- asset characteristics $z_{i,t} \in \mathbb{R}^p$, often high-dim. in practice.
SETUP

Assume for time periods $t = 1, \ldots, T$ and assets $i = 1, \ldots, N$, we observe realizations of random variables

- asset excess returns $r_{i,t+1} \in \mathbb{R}$ and

- asset characteristics $z_{i,t} \in \mathbb{R}^p$, often high-dim. in practice.

Assume the model:

$$r_{i,t+1} = \beta_{i,t}^\top f_{t+1} + \epsilon_{i,t+1},$$

$$\beta_{i,t} = \Gamma_{\beta}^\top z_{i,t} + \epsilon_{i,t},$$
**SETUP**

Assume for time periods $t = 1, \ldots, T$ and assets $i = 1, \ldots, N$, we observe

- asset excess returns $r_{i,t+1} \in \mathbb{R}$ and asset characteristics $z_{i,t} \in \mathbb{R}^p$.

Assume the model:

$$ r_{i,t+1} = z_{i,t}^\top \Gamma_\beta f_{t+1} + \epsilon_{i,t+1}, \quad \mathbb{E} [\epsilon_{i,t+1} | z_{i,t}] = 0, $$

where

- $f_{t+1} \in \mathbb{R}^k$ are low-dimensional latent factors;
- $\Gamma_\beta \in \mathbb{R}^{p \times k}$ are unknown factor loading parameters; and,
- $\epsilon^r_{i,t+1}, \epsilon^\beta_{i,t} \in \mathbb{R}$ are unobserved scalar idiosyncratic errors.
SETUP
Assume for time periods \( t = 1, \ldots, T \) and assets \( i = 1, \ldots, N \), we observe

- asset excess returns \( r_{i,t+1} \in \mathbb{R} \) and asset characteristics \( z_{i,t} \in \mathbb{R}^p \).

Assume the model:

\[
 r_{i,t+1} = z_{i,t}^\top \Gamma \beta_{t+1} + \epsilon_{i,t+1}, \quad \mathbb{E}[\epsilon_{i,t+1}|z_{i,t}] = 0,
\]

where

- \( f_{t+1} \in \mathbb{R}^k \) are low-dimensional latent factors and
- \( \Gamma \beta \in \mathbb{R}^{p \times k} \) are unknown factor loading parameters.

  - Key assump.: \( \Gamma \beta \) is exactly row sparse, i.e. most rows exactly zero.
In this framework, we address a common asset pricing question.
EXTENDED SETUP (1/2)

In this framework, we address a common asset pricing question.

What is the risk premium of an observable nontradable factor \( g_{t+1} \in \mathbb{R} \)?
Within this framework, we address an asset pricing research question.

What is the risk premium of an observable nontradable factor $g_{t+1} \in \mathbb{R}$?

Asset pricing context:

- Risk premium: return for exposure to the factor, ceteris paribus.
- If tradable, the risk premium is the time-series average of the factor.
- If nontradable, form factor mimicking-portfolio.
- Following Giglio, Xiu, and Zhang (2021),
  - assume latent factor model recovers true factor model and
  - project observable nontradable factor onto latent factors.
EXTENDED SETUP (2/2)

What is the risk premium of an observable nontradable factor \( g_{t+1} \in \mathbb{R} \)?

Assume for true latent factors \( f_{t+1} \):

\[
f_{t+1} := \gamma + v_{t+1}, \quad \mathbb{E}[v_{t+1}] = 0
\]

\[
g_{t+1} = \delta + \eta^\top v_{t+1} + \epsilon^g_{t+1}, \quad \mathbb{E}[v_{t+1}\epsilon^g_{t+1}] = 0.
\]
What is the risk premium of an observable nontradable factor $g_{t+1} \in \mathbb{R}$?

Assume for true factors $f_{t+1}$:

$$f_{t+1} := \gamma + \nu_{t+1}, \quad \mathbb{E}[\nu_{t+1}] = 0$$

$$g_{t+1} = \delta + \eta^\top \nu_{t+1} + \epsilon^g_{t+1}, \quad \mathbb{E}[\nu_{t+1} \epsilon^g_{t+1}] = 0.$$  

where

- $\eta \in \mathbb{R}^k$ is an unknown parameter mapping and
- $\epsilon^g_{t+1}$ is measurement error in $g_{t+1}$.  

What is the risk premium of an observable nontradable factor $g_{t+1} \in \mathbb{R}$?

Assume for true factors $f_{t+1}$:

$$f_{t+1} := \gamma + v_{t+1}, \quad \mathbb{E}[v_{t+1}] = 0$$

$$g_{t+1} = \delta + \eta^\top v_{t+1} + \epsilon^g_{t+1}, \quad \mathbb{E}[v_{t+1} \epsilon^g_{t+1}] = 0.$$ 

where

- $\eta \in \mathbb{R}^k$ is an unknown parameter mapping and
- $\epsilon^g_{t+1}$ is measurement error in $g_{t+1}$.

Our target parameter is $\gamma_g = \eta^\top \gamma$. 

EXTENDED SETUP (2/2)
THEORETICAL CONTRIBUTIONS

The model:

\[ r_{i,t+1} = z_{i,t}^\top \Gamma_\beta (\gamma + v_{t+1}) + \epsilon_{i,t+1}, \quad \mathbb{E}[\epsilon_{i,t+1}|z_{i,t}] = 0, \quad \mathbb{E}[v_{t+1}\epsilon_{i,t+1}] = 0, \]
\[ g_{t+1} = \delta + \eta^\top v_{t+1} + \epsilon_{g,t+1}, \quad \mathbb{E}[v_{t+1}\epsilon_{g,t+1}] = 0. \]

Two contributions, under novel asymptotics of \( p, T, N \to \infty \):

1. consistently estimate latent loadings \( \Gamma_\beta \) and factors \( f_{t+1} \) and
2. conduct inference on \( \gamma_g = \eta^\top \gamma \)
   - under novel use of a dynamic latent factor model.
OUTLINE

1. Preview
2. Motivation
3. Setup
4. Theoretical Contributions
5. Theory Literature Review
6. Estimation
7. Key Assumptions
8. Asymptotic Results
9. Proof Outlines
10. Monte Carlo Evidence
11. Empirical Research Questions
12. Empirical Literature Review
13. Empirical Setting
15. Empirical Results: Low Dim. Factor Models
16. Empirical Results: High Dimensional Models
THEORY LITERATURE REVIEW

The scope of the relevant literature is enormous. To name a few:


- *Tests of observable factors*: Fama and MacBeth (1973) Fama-MacBeth, Feng, Giglio, and Xiu (2020) Factor Zoo, Giglio and Xiu (2021), etc.

- *DML*: Belloni, Chernozhukov, and Hansen (2014), Chernozhukov et al. (2018), Semenova and Chernozhukov (2021), etc.
ESTIMATION (1/4)

\[ r_{i,t+1} = z_{i,t}^\top \Gamma_\beta f_{t+1} + \epsilon_{i,t+1}, \quad E[\epsilon_{i,t+1}|z_{i,t}] = 0 \]
Rewrite the model:

\[ r_{i,t+1} = z_{i,t}^\top \Gamma_{\beta} f_{t+1} + \epsilon_{i,t+1}, \quad E[\epsilon_{i,t+1}|z_{i,t}] = 0, \]

\[ = z_{i,t,j} c_{t+1,j} + z_{i,t,-j} c_{t+1,-j} + \epsilon_{i,t+1}, \]

\[ c_{t+1,j} := \Gamma_{\beta,j} f_{t+1}. \]
Rewrite the model:

\[
\mathbf{r}_{i,t+1} = \mathbf{z}_{i,t}^{\top} \Gamma_{\beta} \mathbf{f}_{t+1} + \epsilon_{i,t+1},
\]

\[
= \mathbf{z}_{i,t,j} c_{t+1,j} + \mathbf{z}_{i,t,-j} c_{t+1,-j} + \epsilon_{i,t+1},
\]

\[
E[\epsilon_{i,t+1}|\mathbf{z}_{i,t}] = 0,
\]

\[
c_{t+1,j} := \Gamma_{\beta,j}^{\top} \mathbf{f}_{t+1}.
\]

To estimate \( c_{t+1,j} \forall t,j \)

- run Lasso to account for \( p \sim \mathcal{N} \),
ESTIMATION (1/4)

Rewrite the model:

\[ r_{i,t+1} = z_{i,t}^\top \Gamma_{\beta} f_{t+1} + \epsilon_{i,t+1}, \]

\[ = z_{i,t,j} c_{t+1,j} + z_{i,t,-j} c_{t+1,-j} + \epsilon_{i,t+1}, \quad E[\epsilon_{i,t+1}|z_{i,t}] = 0, \]

\[ c_{t+1,j} := \Gamma_{\beta,j} f_{t+1}. \]

To estimate \( c_{t+1,j} \forall t, j \)

- run Lasso to account for \( p \sim N \), but then biased inference for \( \gamma_g \);
Rewrite the model:

\[ r_{i,t+1} = z_{i,t} \Gamma \beta f_{t+1} + \epsilon_{i,t+1}, \]

\[ = z_{i,t,j} c_{t+1,j} + z_{i,t,-j} c_{t+1,-j} + \epsilon_{i,t+1}, \quad E[\epsilon_{i,t+1}|z_{i,t}] = 0, \]

\[ c_{t+1,j} := \Gamma \beta_j f_{t+1}. \]

To estimate \( c_{t+1,j} \forall t, j \)

- run Lasso to account for \( p \sim N \), but then biased inference for \( \gamma_g \);
- instead run Double Selection Lasso (DSL).
ESTIMATION (2/4)

Model:

\[ r_{i,t+1} = Z_{i,t,j} c_{t+1,j} + Z_{i,t,-j} c_{t+1,-j} + \epsilon_{i,t+1}, \quad E[\epsilon_{i,t+1}|z_{i,t}] = 0, \]
\[ c_{t+1,j} := \Gamma_{\beta,j} f_{t+1}. \]

Procedure:

1. To estimate \( \hat{c}_{t+1,j} \), run \( T \times p \) cross sectional DSL regressions.

2. To estimate \( \hat{\Gamma}_{\beta} \in \mathbb{R}^{p \times k} \) and \( \hat{F} \in \mathbb{R}^{T \times k} \), run PCA on \( \hat{C} := \hat{F} \hat{\Gamma}_{\beta}^\top \in \mathbb{R}^{T \times p} \).

3. Given exact row sparsity, soft-threshold \( \hat{\Gamma}_{\beta} \) to set most rows to zero for \( \tilde{\Gamma}_{\beta} \).
ESTIMATION (3/4)

Model for risk premium of nontradable observable factors:

\[ r_{i,t+1} = z_{i,t}^{\top} \Gamma \beta (\gamma + v_{t+1}) + \epsilon_{i,t+1}, \quad \mathbb{E}[\epsilon_{i,t+1}|z_{i,t}] = 0, \quad \mathbb{E}[v_{t+1}\epsilon_{i,t+1}] = 0, \]

\[ g_{t+1} = \eta^{\top} v_{t+1} + \epsilon_{t+1}^g, \quad \mathbb{E}[\epsilon_{t+1}^{g}] = 0, \quad \mathbb{E}[v_{t+1}\epsilon_{t+1}^{g}] = 0. \]
ESTIMATION (3/4)

Model for risk premia of nontradable observable factors:

\[ r_{i,t+1} = z_{i,t}^\top \Gamma_\beta (\gamma + v_{t+1}) + \epsilon_{i,t+1}, \quad \mathbb{E}[\epsilon_{i,t+1}] = 0, \quad \mathbb{E}[v_{t+1}\epsilon_{i,t+1}] = 0, \]
\[ g_{t+1} = \eta^\top v_{t+1} + \epsilon_{g,t+1}^g, \quad \mathbb{E}[\epsilon_{g,t+1}^g] = 0, \quad \mathbb{E}[v_{t+1}\epsilon_{g,t+1}^g] = 0. \]

Identification:

- Cannot jointly estimate \( \eta \) and \( v_{t+1} \) (\( \Gamma_\beta \) and \( f_{t+1} \)) without further restrictions. E.g., three classic approaches of Bai and Ng (2013).
- So parameters are identified up to rotation matrix \( H \in \mathbb{R}^{k \times k} \). That is, \( \eta = H^{-1}\eta_0 \) and \( \gamma = H\gamma_0 \) (\( \Gamma_\beta = \Gamma_0^b H^{-1} \) and \( f_{t+1} = Hf_{t+1}^0 \)).
- Utilize rotation invariant result of Giglio and Xiu (2021):

\[ \gamma_g = \eta_0^\top H^{-1\top} H\gamma_0 = \eta^\top \gamma \]
ESTIMATION (4/4)

Model for risk premia of nontradable observable factors:

\[ r_{i,t+1} = z_{i,t}^\top \Gamma_\beta (\gamma + v_{t+1}) + \epsilon_{i,t+1}, \quad \mathbb{E}[\epsilon_{i,t+1}] = 0, \quad \mathbb{E}[v_{t+1} \epsilon_{i,t+1}] = 0, \]

\[ g_{t+1} = \eta^\top v_{t+1} + \epsilon_{t+1}^g, \quad \mathbb{E}[\epsilon_{t+1}^g] = 0, \quad \mathbb{E}[v_{t+1} \epsilon_{t+1}^g] = 0. \] (2)

Procedure: \( \hat{\gamma}_g = \hat{\eta}^\top \hat{\gamma} \)

- Estimate factor innovations \( \hat{v}_{t+1} \) and loadings \( \hat{\Gamma}_\beta \) as before but with demeaned returns.
- Estimate latent factor risk premia \( \hat{\gamma} \) via CS OLS of average returns \( \bar{r} \in \mathbb{R}^N \) on estimated latent factor loadings \( \hat{\beta} := T^{-1} \sum_t Z_t \hat{\Gamma}_\beta \in \mathbb{R}^N. \)
- Estimate latent to observable factor mapping \( \hat{\eta} \) via TS OLS of demeaned \( g_{t+1} \) on estimated latent factor innovations \( \hat{v}_{t+1}. \)
OUTLINE

1. Preview
2. Motivation
3. Setup
4. Theoretical Contributions
5. Theory Literature Review
6. Estimation
7. Key Assumptions
8. Asymptotic Results
9. Proof Outlines
10. Monte Carlo Evidence
11. Empirical Research Questions
12. Empirical Literature Review
13. Empirical Setting
15. Empirical Results:
   Low Dim. Factor Models
16. Empirical Results:
   High Dimensional Models
Assumption (Consistency of DSL)

1. Sparse Loading: Loading matrix $\Gamma_\beta$ admits an exactly sparse form. That is, for $\exists s \in \mathbb{N}_+, i.e. p > s \geq 1$, $\Gamma_\beta$ has at most $s$ nonzero rows: $\sum_{j=1}^p 1\left\{ \| \Gamma_{\beta,j} \|_1 > 0 \right\} \leq s$. 

Additional DSL Assumptions
Assumption (Consistency of Latent Factor Model)

2. Nonzero and distinct eigenvalues: from the infeasible eigendecomposition of \((T \ p)^{-1}CC^\top\), the \(k\) largest eigenvalues \(\lambda_i\) for \(i \in \{1, \ldots, k\}\) are bounded away from zero and distinct,

\[
\min_{i: i \neq k} |\lambda_k - \lambda_i| > 0.
\]
Proposition (Consistency of Latent Factors)

Under the DSLFM model (1) and aforementioned Assumptions 1 and 2, with additional Appendix Assumptions 1-6, where $T, N, p \to \infty$, then for all $t$ the latent factor estimator has the property that

$$\hat{f}_{t+1} - H^\top f_{t+1}^0 = O_p \left( \sqrt{s \log(Tp)} \right).$$
Recall $C = F \Gamma_{\beta}^{\top}$, thus $(T p)^{-1} CC^{\top} = (T p)^{-1} F \Gamma_{\beta}^{\top} \Gamma_{\beta} F^{\top}$.

Key rate: $\max_{t,j} |\hat{c}_{t+1,j} - c_{t+1,j}| = O_p \left( \frac{\sqrt{\log(T p)}}{N} \right)$.

Gives control over the distance between feasible and infeasible matrix:

$$\left\| (T p)^{-1} \hat{C} \hat{C}^{\top} - (T p)^{-1} CC^{\top} \right\| = O_p \left( \frac{\log T p}{N} \right).$$

Davis Kahan Theorem bounds distance between eigenvectors by distance between matrices.

Finally, use Wely inequality to bound distance between eigenvalues.
Proposition (Consistency of Latent Factor Loadings)

Under the DSLFM model (1) and aforementioned Assumptions 1 and 2, with additional Appendix Assumptions 1-6, where $T, N, p \to \infty$, then the latent loading estimator has the property that

$$
\tilde{\Gamma}_\beta - \Gamma^0_\beta H^{-1} = O_p \left( \sqrt{\frac{s \log(Tp)}{N}} \right).
$$
PROOF OUTLINE: CONSISTENT LOADINGS

Aforementioned results yield:

\[ \left\| \hat{\Gamma}_{\beta} - \Gamma_0^{\beta} (H^\top)^{-1} \right\|_{\infty} = O_p \left( \sqrt[\beta]{\log(T \rho)} \right). \]

Utilizing Theorem 2.10 from Belloni et al. (2018) under exact sparsity of \( \Gamma_0^{\beta} \), s.t.

\[ \lambda \geq (1 - \alpha) - \text{quantile of } \left\| \hat{\Gamma}_{\beta} - \Gamma_0^{\beta} (H^\top)^{-1} \right\|_{\infty}, \]

then given \( \alpha \to 0 \) and \( \lambda \lesssim \sqrt{\log(T \rho)}/N \), we have for all \( q \geq 1 \)

\[ \left\| \hat{\Gamma}_{\beta,l} - \Gamma_0^{\beta} (H^\top)_{l}^{-1} \right\|_{q} \lesssim_P s^{1/q} \sqrt[\beta]{\log(T \rho)}/N. \]
Theorem (Normality of Observable Factor Risk Premium)

Under the models (1) and (2); Assumptions 1 and 2; Appendix Assumptions 1-10, and, if \( Ts^2 \log(T \cdot p)/N \rightarrow 0 \), then as \( T, N, p \rightarrow \infty \) the estimator \( \hat{\gamma}_g \) obeys

\[
\sqrt{T} \frac{(\hat{\gamma}_g - \gamma_g)}{\sigma_g} \xrightarrow{d} \mathcal{N}(0, 1).
\]
MONTE CARLO EVIDENCE (1/2)

Goal: study the finite-sample estimation error of our latent loading and factor estimators and the coverage properties of our risk premium estimator compared to relevant benchmarks.

DGP: for $S = 200, T = 100, N = 500, k = 3, p \in \{10, 50\}, s = p/10$

- Latent loadings: fit IPCA to empirical panel; set $p - s$ rows to zero.
- Latent factors: fit IPCA to empirical panel; fit VAR(1) to fitted latent factors; simulate from fitted VAR(1) with normal innovations.
- Characteristics: fit panel VAR(1) to demeaned empirical panel of $\{Z_t\}_{t=1}^T$ and simulate from VAR(1) with normal innovations. Set means to bs.
- Returns and observable factor are generated according to the model where errors are calibrated to empirical $R^2$. 
MONTE CARLO EVIDENCE (2/2)

Low-Dimensional: \( p = 10 \)

- Factor of \( \sim 3 \) superior estimation error for \( \Gamma_{\beta} \).
- Order of magnitude inferior estimation error for \( f_{t+1} \).
- DSLFM under-covers (6-9%) while Giglio over-covers (2-4%) \( \gamma_g \).

High-Dimensional: \( p = 50 \)

- Factor of >3 superior estimation error for \( \Gamma_{\beta} \).
- Inferior (\( \times 4 \)) estimation error for \( f_{t+1} \).
- DSLFM degrades 1% while Giglio degrades > 3% \( \gamma_g \).
EMPIRICAL RESEARCH QUESTIONS

_Broadly:_ Study the dynamics of crypto asset excess returns.

_Specifically:_

- Measuring expected returns through lens of factor models.
- What characteristics are the drivers of returns?
- What is the inflation risk premium in the crypto asset class?
- If we relax interpretability, what is the maximum out-of-sample predictability that can we achieve?
EMPIRICAL LITERATURE REVIEW

Empirical crypto asset pricing is a nascent literature. To name a few:

- **Asset pricing ability of factors models**: Liu et al. (2019), Shams (2020), Bianchi and Babiak (2021), Liu, Tsyvinski, and Wu (2022), etc.


- **Crypto panel**: Liebi (2022), Borri et al. (2022), Cong et al. (2022).

EMPIRICAL SETTING

- Weekly panel of crypto asset excess returns from 2018-2022, inclusive, with 63 time-varying asset characteristics. (Summary Stats. Most go to zero.)

- Inclusion criteria: month by month look back over trailing 3 months
  - tradable on US CEX;
  - remove stablecoins and synthetic assets;
  - asset mean mcap above 1 bps of total crypto mcap; and,
  - asset median total weekly trade volume on US exchanges above $500k.

- When fitting models sequentially, have to reform panel monthly.

- Form price from volume-weighted average hourly candle mid price.
OUTLINE

1. Preview
2. Motivation
3. Setup
4. Theoretical Contributions
5. Theory Literature Review
6. Estimation
7. Key Assumptions
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9. Proof Outlines
10. Monte Carlo Evidence
11. Empirical Research Questions
12. Empirical Literature Review
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Crypto Signals:

1. There are several characteristics with significant signal for the cross-section of one-week ahead expected returns.

2. The asset characteristics contain redundant information; however, the variation cannot be captured by just a few principal components.

Figures: Char. Correlations and Signal.
A New and Rising Asset Class:

3. From zero in 2009, Bitcoin and hundreds of other crypto assets have become a trillion dollar asset class in 2022, with several multi-billion dollar sub-industries.

4. Bitcoin achieved superior risk-adjusted returns for nearly the entire study time period as compared to traditional asset classes.

5. Bitcoin has lower correlations to the Nasdaq and S&P500 (at 0.23 and 0.21) than that of gold’s correlation to these indices (at 0.26 and 0.28).

6. Bitcoin’s correlation with other assets is highly time varying, including several quarters of zero or negative correlation with the Nasdaq; their high correlation (> 0.3) is only observed recently in 2022.

7. From diversifying a risk portfolio of holding 100% Nasdaq to 60% Nasdaq and 40% CMKT, one would obtain a Sharpe Ratio gain of 0.53 (from 0.43 to 0.96).

8. The crypto market offers a positive inflation risk premium of 31 bps.
Bitcoin Onchain Facts:

10. Bitcoin is primarily used as a store value, not speculatively trading.

11. Bitcoin is a payment network settling hundreds of billions of dollars annually where the large majority of transactions cost less than one USD.

12. Efforts to fork, that is copy, the Bitcoin blockchain have had immaterial adverse effects on it; an event study of forks observes, on the contrary, significant positive effects on price, trading volume, active addresses, and social activity.

Figures: [BTC Hodling], [BTC Tx], [BTC Forks].
• Begin our empirical study of return dynamics by forming univariate factors from each asset characteristic.

• Form zero-net investment long-short quintile strategies for $t + 1$ sorted on each characteristic at time $t$.

• Statistically and economically significant strategies are financial (i.e. two week momentum, 30 and 60 day industry momentum, beta, idiosyncratic skewness, and 5% shortfall).

• 35 of 57 remaining strategies have weekly excess return above 30 bps.

• Predictors of average excess returns appear to be characteristic-based factors formed as functions of previous returns.
LOW DIM. MODELS: MULTI, PCA, IPCA (2/2)

- Multifactor models: static observable, static latent, & dynamic latent.
- Estimation procedures are Fama-MacBeth, PCA, and IPCA.
- All three have long-short strategies with economically significant Sharpe ratios of 1-4 (IPCA has stat. sig.) although optimal number of factors is inconsistent.
- Replicates IPCA to new asset class, and suggestive of signal in characteristics.
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1. Preview
2. Motivation
3. Setup
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5. Theory Literature Review
6. Estimation
7. Key Assumptions
8. Asymptotic Results
9. Proof Outlines
10. Monte Carlo Evidence
11. Empirical Research Questions
12. Empirical Literature Review
13. Empirical Setting
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Study three questions:

1. out of sample predictability;
2. characteristic importance; and,
3. inflation risk premium.
In comparing predictability of DSLFM to previous models:

- CV penalty hyperparameters in Q3 2021 - Q2 2022.

- Positive $R^2_{pred}$ for only 1 factor model.

- Sharpe ratios of $\sim 1$ for mcap weighted and $\sim 2.5$ for equal weighted.

- IPCA outperforms on Sharpe and pricing ability, but not materially.

- Limited $N$ for asymptotics to kick in; no feature selection for feat. imp.
• Implement bootstrap procedure for characteristic importance:
\[
\hat{W}_j = \hat{\Gamma}_{\beta,j}^\top \hat{\Gamma}_{\beta,j}.
\]

• According to the DSLFM, the drivers of returns are exchange inflows and outflows.
Outstanding question on the relationship of crypto’s returns to inflation.

Perform inference procedure for inflation risk premium $\gamma_g$.

Inflation risk premium of statistically significant 1.4 bps per week.

Corroborates with a dynamic latent factor model with superior pricing ability the result using the static observable factor model fit with the Fama-MacBeth procedure.

The asset class provides investors positive compensation for holding an inflation-hedged crypto portfolio, ceteris paribus.
We ask:

• what is the maximum out of sample Sharpe ratio achievable
• in our out of sample period
• by relaxing interpretability of the factor model to specify nonparametric factors and factor loadings
• estimated with deep learning?
HIGH DIM. MODELS: DEEP LEARNING (2/6)

Why DL to learn $f(\cdot)$ in $r_{i,t+1} = f(l_t) + \epsilon_{i,t+1}$:

- $l_t$ is high dimensional
- $l_t$ redundant $\implies$ regularization!
- $f(\cdot)$ is likely non-linear.
- DL’s universal approximation theorems, e.g. Hornik, Stinchcombe, and White (1990).
- Minuscule improvement is the name of the game.
HIGH DIM. MODELS: DEEP LEARNING (2/6)

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- DL’s universal approximation theorems, e.g. Hornik, Stinchcombe, and White (1990).
- Minuscule improvement is the name of the game.
HIGH DIM. MODELS: DEEP LEARNING (3/6)

Now, why not?

\[ r_{i,t+1} = f(l_t) + \epsilon_{t+1}, \quad E[\epsilon_{t+1}|l_t] = 0. \]

- EMH \implies f(\cdot) = 0, such that \( r_{i,t+1} = 0 + \epsilon_{i,t+1} \).
- It’s \( f_t(\cdot) \), not \( f(\cdot) \).
- Makes the curse of dimensionality much bigger problem.
- Perhaps SOTA DL with lots of regularization?
-high dim. models: deep learning (3/6)

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HIGH DIM. MODELS: DEEP LEARNING (3/6)

Now, why not?

\[
\begin{align*}
    r_{i,t+1} &= f(l_t) + \epsilon_{t+1}, \\
    E[\epsilon_{t+1}|l_t] &= 0.
\end{align*}
\]

- \( EMH \implies f(\cdot) = 0 \), such that \( r_{i,t+1} = 0 + \epsilon_{i,t+1} \).
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HIGH DIM. MODELS: DEEP LEARNING (3/6)

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- Makes the curse of dimensionality much bigger problem.
- Perhaps SOTA DL with lots of regularization?
HIGH DIM. MODELS: DEEP LEARNING (4/6)
Utilize recent adaption of feed-forward neural networks within a factor model structure: stay true to equilibrium asset pricing theory.

Several modifications:

- Nonlinear factor autoencoder.
- Sequential CV with more hp points: learning rate decay, Adam parameters, weights and bias initializers, etc.
- Weight loss by hourly trade volume over previous hour.
- Feature selection down to 50 characteristics.

Also, embed Transformers into factor model structure.
Iterative step-forward CV: for each hyperparameter point in the grid,

- Set training data to 2018-2020.
  - For transformer, dropped 2018-2019 due to too many missing assets.
  - 2020 has 532,368 non-missing asset-hours.
- Set validation data to Q1 2021.
- For each week in the validation period,
  - Fit in training and predict in val week.
- Add current validation data to training, set next quarter to validation.

Select best model from validation period. OOS predict, once.
Autoencoder dominates on out of sample Sharpe at 10.

- It failed to achieve positive out-of-sample predictive $R^2$.

Transformer had a stat. sig. $R^2_{pred}$ of 3.6%,

- But, returns were negative in the Q3-Q4 2022 data to transaction costs,
- which motivates further research with 2023 data.
- Scaling laws present in validation period up to 50-100k parameters,
  but did not replicate out of sample.

I am focused on exploring these DL scaling laws in practice.
REFERENCES


Borri, Nicola, Daniele Massacci, Mirco Rubin, and Dario Ruzzi. 2022. “Crypto risk premia.” Available at SSRN.


Cong, Lin William, George Andrew Karolyi, Ke Tang, and Weiyi Zhao. 2022. “Value premium, network adoption, and factor pricing of crypto assets.”


Thank You!
APPENDIX: IPCA

The model is

\[ r_{i,t} = Z_{i,t-1} \Gamma_{\delta} f_t + \epsilon_{i,t}. \]

The objective function is to minimize the sum of the squared errors:

\[ \min_{\Gamma_{\delta}, f_t} \sum_{t=1}^{T} (r_t - Z_{t-1} \Gamma_{\delta} f_t)^	op (r_t - Z_{t-1} \Gamma_{\delta} f_t). \]
The first-order conditions are

\[
\hat{f}_t = \left( \hat{\Gamma}_\delta' Z_{t-1}' Z_{t-1} \hat{\Gamma}_\delta \right)^{-1} \hat{\Gamma}_\delta' Z_{t-1}' r_t,
\]

\[
\text{vec} \left( \hat{\Gamma}'_\delta \right) = \left( \sum_{t=1}^{T-1} Z_{t-1}' Z_{t-1} \otimes \hat{f}_t \hat{f}_t' \right)^{-1} \left( \sum_{t=1}^{T-1} \left[ Z_{t-1} \otimes \hat{f}_t \right]' r_t \right)
\].

Factor realizations are period-by-period cross section regression coefficients of \(r_t\) on the latent loading matrix \(\delta_{t-1}\).

\(\Gamma_\delta\) is the coefficient of returns regressed on the factors interacted with firm-specific characteristics.
APPENDIX: IPCA

Similarities:

(Second-stage) factor model relationship and joint fitting.
Cross-sectional and time-series two step procedures a la Fama MacBeth.
Efficiency gains from using asset covariates.
Accommodate unbalanced panels.

Pro Double Lasso:

Sparse estimation
Convex objective functions
Model high dimensional $p$
Closed-form inference for target question

Pro IPCA:

Conceptually simpler optimization
Fewer assumptions for asymptotic theory
Rapid estimation
APPENDIX: FAMA-MACBETH REGRESSIONS

The classic observable factor model estimation is the Fama and MacBeth (1973) procedure.

We first run $N$ TS regressions for each asset followed by $T$ CS regressions for each time period.

That is, we first estimate $\hat{\beta}_i$ for each asset $i$ by running TS OLS of $\{r_{i,t+1}\}_{t=1}^T$ on $\{f_{t+1}\}_{t=1}^T$.

Next, we run $\forall t$ the CS OLS of asset excess returns $\{r_{i,t+1}\}_{i=1}^N$ on estimated factor loadings $\{\hat{\beta}_i\}_{i=1}^N$.

We recover estimates $\hat{\lambda}_t$ for the risk premium $\lambda_t = \mathbb{E}_t[f_{t+1}]$ as well as the pricing errors from the cross-sectional residuals, $\hat{\alpha}_{i,t+1}$.

Finally, we estimate the parameters of interest: the static risk premium $\hat{\lambda}$ and the static average pricing error $\hat{\alpha}_i$ as the time-series averages of the relevant estimator, $\hat{\lambda}_t$ and $\hat{\alpha}_{i,t+1}$, respectively.
APPENDIX: DSL ESTIMATION PROCEDURE

\[ r_{i,t+1} = Z_{i,t,j}c_{t+1,j} + Z_{i,t,-j}^\top c_{t+1,-j} + \epsilon_{i,t+1}, \quad E[\epsilon_{i,t+1}|z_{i,t}] = 0, \]

\[ z_{i,t,j} = Z_{i,t,-j}^\top \delta_{t,j} + \epsilon_{i,t,j}^Z, \quad E[\epsilon_{i,t,j}^Z|z_{i,t,-j}] = 0, \]

\[ c_{t+1,j} := \Gamma^\top_{\beta,j} f_{t+1}. \]

For \( \hat{c}_{t+1,j} \), run \( T \times p \) Double Selection Lasso CS regressions \( \forall t, j. \)

\[
\text{Lasso } \left\{ r_{i,t+1} \right\}_{i=1}^N \rightarrow \left\{ z_{i,t} \right\}_{i=1}^N \text{ for } \hat{l}_1 = \text{nonzero elements of } \hat{c}_t.
\]

\[
\text{Lasso } \left\{ z_{i,t,j} \right\}_{i=1}^N \rightarrow \left\{ z_{i,t,-j} \right\}_{i=1}^N \text{ for } \hat{l}_2 = \text{nonzero elements of } \hat{\delta}_{t,j}.
\]

Define \( \hat{l} := \hat{l}_1 \cup \hat{l}_2 \cup \hat{l}_3 \) where \( \hat{l}_3 \) is manually chosen.

\[
\text{OLS } \left\{ r_{i,t} \right\}_{i=1}^N \text{ on elements of } \left\{ z_{i,t-1} \right\}_{i=1}^N \text{ in } \hat{l}.
\]
Assumption (DSL Uniform Consistency)

1. Bounded Characteristic Portfolios: For a finite absolute constant $M$ and $\forall t, j,$

   \[ |c_{t+1,j}| = |\Gamma_{\beta,j}^T f_{t+1}| < M. \]

2. Sparsity rate: The sparsity index obeys

   \[ s^2 \log^2 (p \vee N) / \left( \sqrt{N \log(T p)} \right) \leq \delta_{N,T}. \]

   Additionally, \( \log^3 p/N \leq \delta_{N,T}. \)

3. Weak dependence between the first- and second-stage errors: There exists a positive constant $M$ such that $\forall p, T, N$

   \[
   \left| \sqrt{\frac{1}{N}} \sum_{i=1}^{N} \epsilon_{i,t,j}^Z \epsilon_{i,t+1} \right| \leq M \log(T p).
   \]

4. Additional standard DSL assumptions in Appendix C.2 of the paper.
APPENDIX: ASSUMPTIONS

Assumption (Consistency of Latent Factor Model)

5. **Factors:** $\mathbb{E} \left\| f_{t+1}^0 \right\|^4 \leq M < \infty$ and $T^{-1} \sum_t f_{t+1}^0 f_{t+1}^{0\top} \rightarrow_p \Sigma_f$ for some $k \times k$ positive definite matrix $\Sigma_f$.

6. **Factor Loadings:** $\forall j$, $\left\| \Gamma_{\beta,j} \right\| \leq M < \infty$ and $\left\| \Gamma_{\beta}^{\top} \Gamma_{\beta} / p - \Sigma_{\Gamma} \right\| \rightarrow 0$ for some $k \times k$ positive definite matrix $\Sigma_{\Gamma}$. 
APPENDIX: ASSUMPTIONS

Assumption (Inference)

∃ a generic absolute constant $M < \infty$ such that for all $p, T, N$:

7. **Bounded idiosyncratic errors:** $\mathbb{E}[(\sum_t \epsilon_{i,t+1})^2] \leq TM$.

8. **Bounded scaled factor innovations:** $\mathbb{E}[(\sum_t z_{i,t}^\top \Gamma^0_{\beta} v^0_{t+1})^2] \leq sTM$.

9. **Bounded measurement errors:** $\mathbb{E}[(\epsilon_{t+1}^g)^2] \leq M$. 
Assumption (Inference)

9. Convergence of characteristics:

\[
\frac{1}{NT} \sum_i \sum_{t'} \mathbb{E}[z_{i,t,j}z_{i,t',j'}] \to p \quad \forall t, j, j' \text{ uniformly over } t, j, j' \text{ for } j, j' \in \{1, 2, \ldots, p\} \text{ and a nonstochastic finite constant } \exists \in \mathbb{R}.
\]

10. CLT: As \( T \to \infty \),

\[
\frac{1}{\sqrt{T}} \sum_t \begin{pmatrix} v_{t+1}^0 e_{t+1}^g \\ \Pi_t v_{t+1}^0 \end{pmatrix} \overset{d}{\to} \mathcal{N}(0, \Phi)
\]

for random matrix \( \Pi_t \in \mathbb{R}^{k \times k} \) and nonstochastic matrix \( \Phi \in \mathbb{R}^{2k \times 2k} \).
### APPENDIX: SIMULATION LOW-DIMENSIONAL

<table>
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<th>(2) Three-Pass Est.</th>
<th>(3) DSLFM</th>
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Back to Simulation Result Summary.
## APPENDIX: SIMULATION HIGH-DIMENSIONAL

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Back to Simulation Result Summary.
## APPENDIX: SUMMARY STATISTICS

### Panel A. Panel summary by year.

<table>
<thead>
<tr>
<th>Year</th>
<th>Unique Assets</th>
<th>CMKT Excess Return</th>
<th>Total Mcap ($B)</th>
<th>Median Mcap ($B)</th>
<th>Median Volume ($MM)</th>
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<tbody>
<tr>
<td>2018</td>
<td>10</td>
<td>-71.04%</td>
<td>$102</td>
<td>$8.72</td>
<td>$10.27</td>
</tr>
<tr>
<td>2019</td>
<td>15</td>
<td>62.89%</td>
<td>$163</td>
<td>$3.70</td>
<td>$11.96</td>
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<tr>
<td>2020</td>
<td>25</td>
<td>280.61%</td>
<td>$618</td>
<td>$2.05</td>
<td>$11.64</td>
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<tr>
<td>2021</td>
<td>154</td>
<td>332.54%</td>
<td>$2,121</td>
<td>$1.42</td>
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<td>2022</td>
<td>204</td>
<td>-64.05%</td>
<td>$629</td>
<td>$0.45</td>
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<tr>
<td>All</td>
<td>210</td>
<td>179.16%</td>
<td>$629</td>
<td>$0.84</td>
<td>$17.58</td>
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### Panel B. Summary statistics of annualized excess returns.

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<th>Skewness</th>
<th>Kurtosis</th>
<th>% &gt; 0</th>
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<td>0.02</td>
<td>0.53</td>
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<td>0.02</td>
<td>0.52</td>
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<td>Ethereum</td>
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<td>100.11%</td>
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<td>0.03</td>
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### Panel C. Extreme events of weekly CMKT excess returns.

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APPENDIX: MOST ASSETS HAVE NEGATIVE RETURN
APPENDIX: SHARPE RATIOS

Back to Motivating Empirical Facts
## APPENDIX: CORRELATIONS

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Back to Motivating Empirical Facts
APPENDIX: ROLLING CORRELATIONS

![Graph showing rolling correlations over time with different financial and commodity indices](image)

- **Nasdaq**
- **US Bonds**
- **US Real Estate**
- **Emerging Currencies**
- **Commodities**
- **Gold**
- **EXPINF1YR**

Back to Motivating Empirical Facts
APPENDIX: RISK AND RETURN
## Appendix: Inflation Risk Premium

**Panel A. BTC Return Time-Series Regression.**

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**Panel B. Fama-MacBeth Regression.**

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APPENDIX: BITCOIN HODLING: UTXO MEDIAN AGE
APPENDIX: BITCOIN FORK: EVENT STUDY

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APPENDIX: CHARACTERISTIC CORRELATIONS AND SIGNAL

See paper appendix figures A15 through A24.

Back to Motivating Empirical Facts
## APPENDIX: UNIVARIATE FACTOR RETURNS

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Back to Univariate Factor Models.
APPENDIX: LOW DIMENSIONAL FACTOR MODELS

See paper appendix table A31.

Back to Low Dimensional Factor Models.
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Back to DSLFM.
## APPENDIX: DSLFM CHARACTERISTIC IMPORTANCE

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APPENDIX: DL AUTOENCODER

Back to DL Models.
APPENDIX: TRANSFORMER FACTOR MODEL
APPENDIX: DL FM OOS PORTFOLIO STATISTICS

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